

SE Electrical  
S.E. EXTC

Part - III

III CBGS

2015-16  
QP Code : 4787

(3 Hours)  
[ Revised Course ]

[Total Marks : 80]

- N.B.: 1) Question No.1 is compulsory.  
2) Attempt any three from the remaining questions.  
3) Assume suitable data if necessary.

1. (a) Determine the constants a,b,c,d if  $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$  is analytic. 5
- (b) Find a cosine series of period  $2\pi$  to represent  $\sin x$  in  $0 \leq x \leq \pi$  5
- (c) Evaluate by using Laplace Transformation  $\int_0^{\infty} e^{-3x} i \cos t dt$ . 5
- (d) A vector field is given by  $\vec{F} = (x^2 + xy^2) i + (y^2 + x^2 y) j$ . Show that  $\vec{F}$  is irrotational and find its scalar potential. Such that  $\vec{F} = \nabla \phi$ . 5
2. (a) Solve by using Laplace Transform 6  
 $(D^2 + 2D + 5)y = e^{-t} \sin t$ , when  $y(0) = 0$ ,  $y'(0) = 1$ .
- (b) Find the total work done in moving a particle in the force field  $\vec{F} = 3xy i - 5z j + 10x k$  along  $x=t^2 + 1$ ,  $y = 2t^2$ ,  $z=t^3$  from  $t=1$  and  $t=2$ . 6
- (c) Find the Fourier series of the function  $f(x) = e^{-x}$ ,  $0 < x < 2\pi$  and  $f(x+2\pi) = f(x)$ . Hence deduce that the value of  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+1}$ . 8
- 3 (a) Prove that  $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$  6
- (b) Verify Green's theorem in the plane for  $\oint (x^2 - y) dx + (2y^2 + x) dy$  6  
Around the boundary of region defined by  $y = x^2$  and  $y = 4$ .
- (c) Find the Laplace transforms of the following. 8

i)  $e^{-t} \int_0^t \frac{\sin u}{u} du$

ii)  $t \sqrt{1 + \sin t}$

[ TURN OVER

$\left[ \begin{array}{l} \text{S.E. Extension} \\ \text{S.E. EXTC} \end{array} \right]$   
III CBGS

A.M. III  
 2

2015

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4 (a) If  $f(x) = C_1 Q_1(x) + C_2 Q_2(x) + C_3 Q_3(x)$ , where  $C_1, C_2, C_3$  constants and  $Q_1, Q_2, Q_3$  are orthonormal sets on  $(a,b)$ , show that  $\int_a^b [f(x)]^2 dx = c_1^2 + c_2^2 + c_3^2$ . 6

(b) If  $v = e^x \sin y$ , prove that  $v$  is a Harmonic function. Also find the corresponding harmonic conjugate function and analytic function. 6

(c) Find inverse Laplace transforms of the following. 8

i)  $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$       ii)  $\frac{s+2}{s^2-4s+13}$

5 (a) Find the Fourier series if  $f(x) = |x|$ ,  $-k < x < k$  6

Hence deduce that  $\sum \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$

(b) Define solenoidal vector. Hence prove that  $\vec{F} = \frac{\vec{a} \times \vec{r}}{r^n}$  is a solenoidal vector 6

(c) Find the bilinear transformation under which  $i, 1, -1$  from the  $z$ -plane are mapped onto  $0, 1, \infty$  of  $w$ -plane. Further show that under this transformation the unit circle in  $w$ -plane is mapped onto a straight line in the  $z$ -plane. Write the name of this line. 8

6 (a) Using Gauss's Divergence Theorem evaluate  $\iint_S \vec{F} \cdot d\vec{s}$  where  $\vec{F} = 2x^2y\vec{i} - y^2\vec{j} + 4xz^2\vec{k}$  and  $S$  is the region bounded by  $y^2 + z^2 = 9$  and  $x = 2$  in the first octant. 6

(b) Define bilinear transformation. And prove that in a general, a bilinear transformation maps a circle into a circle. 6

(c) Prove that  $\int x^{2/3} (x^{3/2}) dx = -\frac{2}{3} x^{-1/2} ]_{1/3} (x^{3/2})$ . 8